

Problem set 2.: Rigid body motion

Problem 1 Determine the matrix that describes a rotation around the $\mathbf{n} = (0, 1/2, \sqrt{3}/2)$ axis by 60 degree.

Problem 2 Calculate

$$\sum_{k=1}^3 \epsilon_{ijk} \epsilon_{klm} \quad (1)$$

for arbitrary i, j, l, m .

Problem 3 For what value of λ has the system of equations

$$\begin{aligned} 5x + 2y - 3z &= 0 \\ 3x - 2y &= 0 \\ 4x + 3y + \lambda z &= 0 \end{aligned} \quad (2)$$

nontrivial solution?

Problem 4 For which α and β is the product(s) of

$$A = \begin{pmatrix} 1 & 2 & \alpha \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{pmatrix} \quad (3)$$

$$B = \begin{pmatrix} 1 & -1 & 1 \\ -2 & \beta & -1 \\ -2 & 1 & 0 \end{pmatrix} \quad (4)$$

equal to zero?

Problem 5 Calculate the determinant of

$$A = \begin{pmatrix} x & a & a & \dots & a \\ a & x & a & \dots & a \\ \cdot & \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \cdot & \dots & \cdot \\ a & a & a & \dots & x \end{pmatrix} \quad (5)$$

Problem 6 Calculate

$$\frac{d \det(A)}{da_{ij}} \quad (6)$$

where

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ \cdot & \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \cdot & \dots & \cdot \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{pmatrix} \quad (7)$$

Problem 7 Prove that if A and B are $n \times n$ matrices and I is the $n \times n$ unit matrix then

$$AB - BA = I \quad (8)$$

can never be true.

Problem 8 Prove that

$$\sin(A\phi) = A\sin(\phi) \quad (9)$$

where

$$A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad (10)$$

Problem 9 Calculate the inertia matrix for the following two dimensional objects

- a) half circle
- b) isoscale triangle

Determine the principal moments and principal axis!

Problem 10 Calculate the inertia matrix for the the following objects (use the symmetry axis as the z axis of the coordinate system)

- a) sphere
- b) cone
- c) cylinder
- d) thin rod
- e) rectangular parallelepiped
- d) ellipsoid

Problem 11 Calculate the inertia matrix for the the following objects:

- a) triangular molecule
- b) tetratomic molecule (equilateral-based tetrahedron)

Problem 12 Calculate the inertia matrix for the following objects

- a) equilateral triangle formed by thin rods
- b) square formed by thin rods

Problem 13 What is the equation of motion of a compound pendulum (a rigid body swinging about a fixed horizontal axi in a gravitational field).

Problem 14 Two thin rod hinged together like a ladder What is the equation of motion?

Problem 15 What is the equation of motion of a cylinder rolling on an incline?

Problem 16 What is the equation of motion of a ball of radius a rolling inside of a cylindrical surface?

Problem 17 Find the equation of motion for a cone rolling on a plane.

Problem 18 Find the equation of motion for a cone, whose base rolls on a plane and whose vertex is fixed at a height above the plane equal to the radius of the base, so the cone's axis is parallel to the plane.

Problem 19 Find the kinetic energy of an ellipsoid which rotates about one of its axis.

Problem 20 A cylinder rolls inside of a cylinder. The outer cylinder can roll on a plane. Derive the equations of motion.