Homework 3

1. (10 points) A particle moves in the xy-plane subject to the Lagrangian

$$L = \frac{1}{2}(\dot{x}^2 + \dot{y}^2) + \frac{\Omega}{2}(-\dot{x}y + x\dot{y})$$

where ω is a constant.

(a) Write down the Lagrangian equations of motion.

(b) Show that the z-component, J, of angular momentum about the origin is not (in general) conserved.

(c) Show, by contrast that the z-component of the *generalised* angular momentum is conserved.

(d) Show that, for any solution of the equations of motion, there is a fixed point (call it A) such that the z-component of angular momentum, J, about A is conserved.

(e) Find the energy for the system and show that it is conserved.

2. (15 points) Excercise 6 (Goldstein, p. 64)

The gravitational potential inside the earth is

$$V(r) = \begin{array}{ll} \frac{GMm}{2R^3}r^2 - \frac{3}{2}\frac{GMm}{R} & r < R \\ -\frac{GMm}{r} & R < r \end{array} \tag{1}$$

(M is the mass, R is the radius of the earth)

3. (5 points) Calculate the functional derivative of

$$I(y) = \frac{1}{2}\int \left(\frac{dy}{dx}\right)^2 dx' \tag{2}$$