XI. EIGENVALUE PROBLEMS

Eigenvalue problem: $\underline{A}$ is a $n \times n$ matrix,

$$\underline{A}\underline{x}_i = \lambda_i \underline{x}_i$$

λ_i $(i = 1, ..., n)$ are the eigenvalues and $\underline{x}_i$ are the eigenvectors of $\underline{A}$. The eigenvectors are orthogonal:

$$\sum_{k=1}^{n} x_{ik} x_{jk} = \delta_{ij}$$

The matrix $\underline{X}$

$$\underline{X} = \begin{pmatrix} x_{11} & x_{21} & ... & x_{n1} \\ x_{12} & x_{22} & ... & x_{n2} \\ . & . & ... & . \\ x_{1n} & x_{2n} & ... & x_{nn} \end{pmatrix}$$

constructed from the eigenvectors, diagonalizes $\underline{A}$

$$\underline{X}^T \underline{A}\underline{X} = \begin{pmatrix} \lambda_1 & 0 & ... & 0 \\ 0 & \lambda_2 & 0 & 0 \\ . & . & ... & . \\ 0 & . & ... & \lambda_n \end{pmatrix}$$

Generalized Eigenvalue problem: $\underline{A}$ and $\underline{B}$ is a $n \times n$ matrix,

$$\underline{A}\underline{x}_i = \lambda_i \underline{B}\underline{x}_i$$

λ_i $(i = 1, ..., n)$ are the eigenvalues and $\underline{x}_i$ are the eigenvectors of $\underline{A}$. The eigenvectors are orthogonal with respect to $\underline{B}$

$$\sum_{k,l=1}^{n} x_{ik} B_{kl} x_{jl} = \delta_{ij}$$

and

$$\sum_{k,l=1}^{n} x_{ik} A_{kl} x_{jl} = \delta_{ij} \lambda_i$$