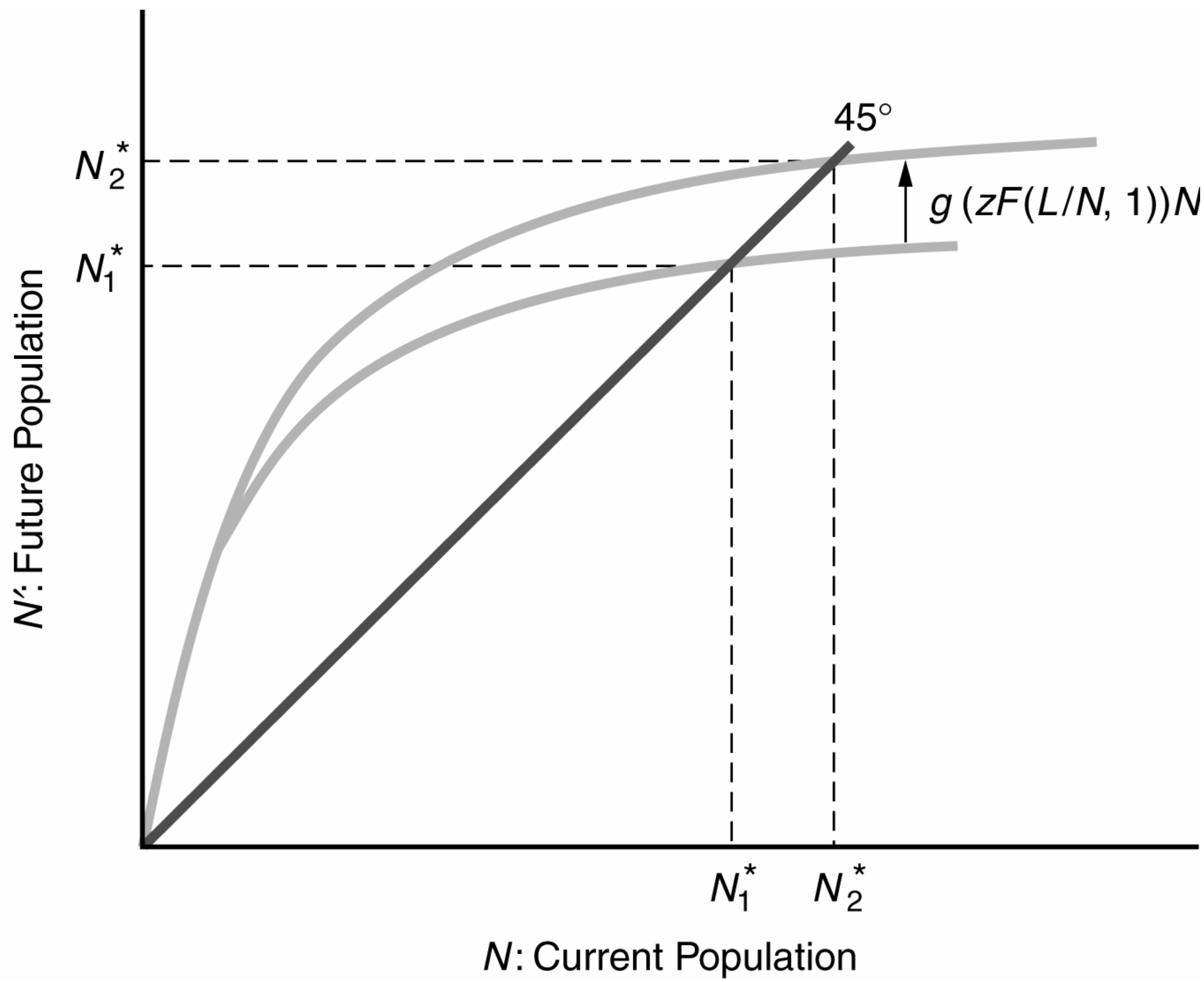
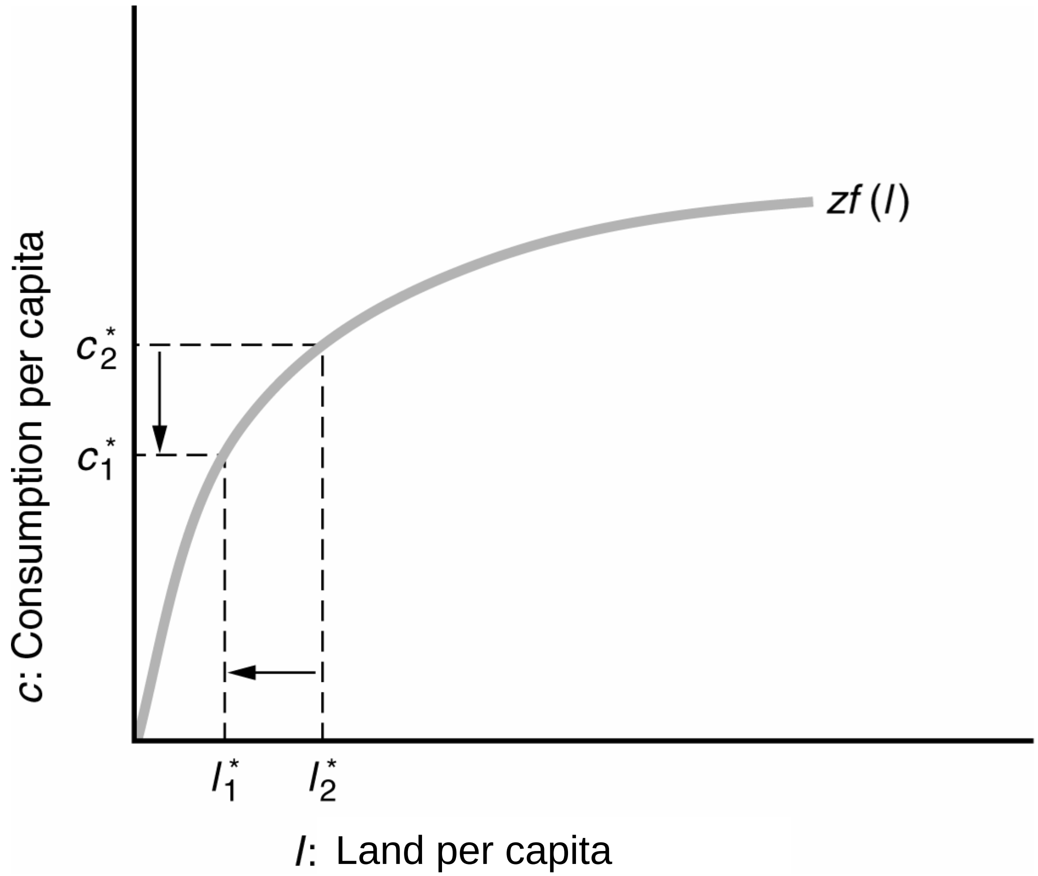
Chapter 6, Question #1

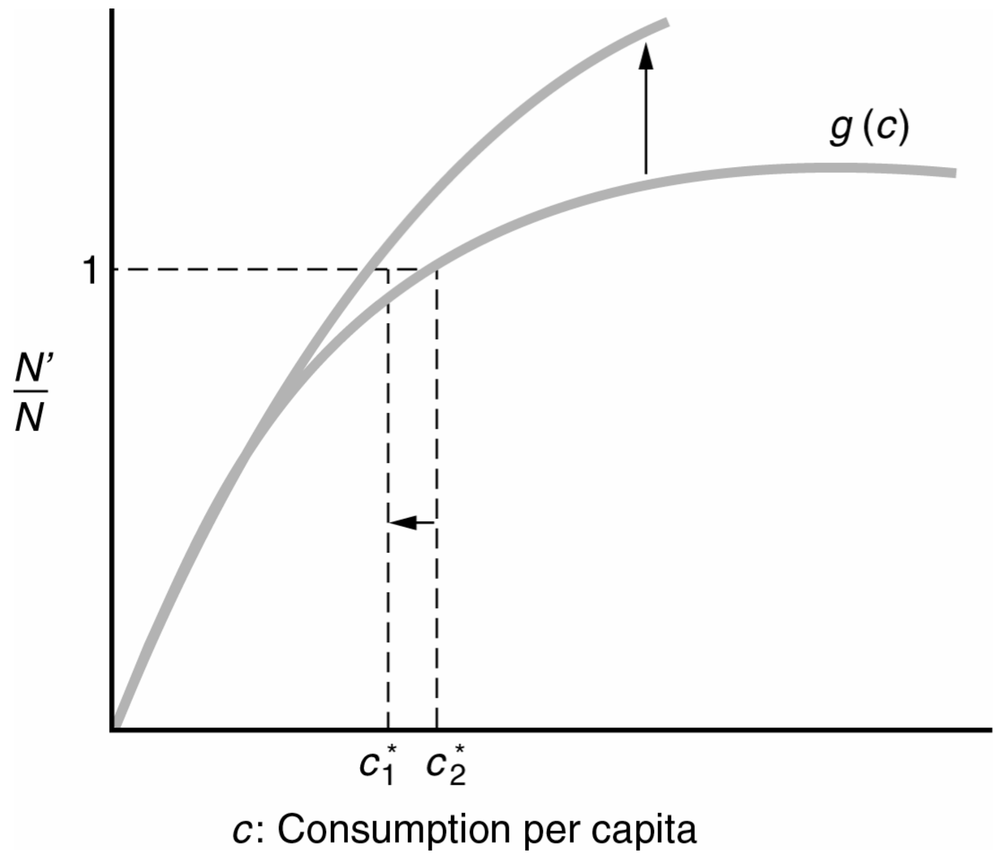
The amount of land increases, and, at first, the size of the population is unchanged. Therefore, consumption per capita increases. However, the increase in consumption per capita increases the population growth rate, see the figure below. In the steady state, neither c^* nor l^* are affected by the initial increase in land. This fact can be discerned by noting that there will be no changes in either of the panels of Figure 6.8 in the textbook.


45°
N_2^*
N_1^*
$g\,(zF(L/N,\,1))N$
N': Future Population
N_1^*
N_2^*
N: Current Population

Chapter 6, Question #2

A reduction in the death rate increases the number of survivors from the current period who will still be living in the future. Therefore, such a technological change in public health shifts the function $g(c)$ upward. In problem #1 there were no effects on the levels of land per capita and consumption per capita. In this case, the $g(c)$ function in the bottom figure below shifts upward. Equilibrium consumption per capita decreases. From the top figure below, we also see that the decrease in consumption per capita requires a reduction in the equilibrium level of land per capita. The size of the population has increased, but the amount of available land is unchanged.

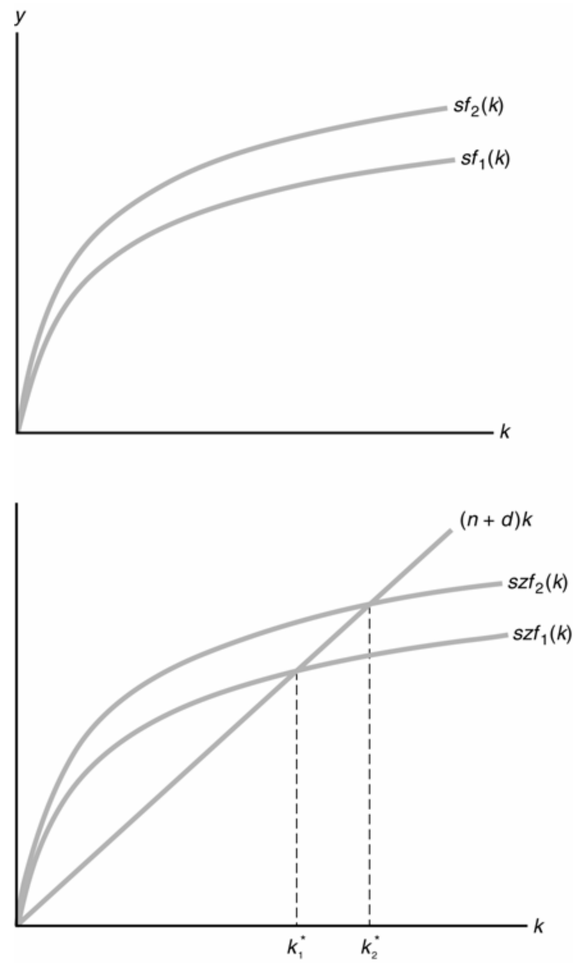

zf (l)
c: Consumption per capita
c_2^*
c_1^*
l_1^*
l_2^*
l: Land per capita


g (c)
1
N'/N
c_1^*
c_2^*
c: Consumption per capita

Chapter 6, Question #3

For the marginal product of capital to increase at **every** level of capital, the shift in the production function is equivalent to an increase in total factor productivity.

(a) The original and new production functions are depicted in the figures below.


y
sf2(k)
sf1(k)
k
(n + d)k
szf2(k)
szf1(k)
k
k1*
k2*

(b) Equilibrium in the Solow model is at the intersection of $szf(k)$ with the line segment $(n+d)k.$ The old and new equilibria are depicted in the bottom panel of the figure above. The new equilibrium is at a higher level of capital per capita and a higher level of output per capita.

(c) For a given savings rate, more effective capital implies more savings, and in the steady state there is more capital and more output. However, if the increase in the marginal product of capital were local, in the neighborhood of the original equilibrium, there would be no equilibrium effects. A twisting of the production function around its initial point does not alter the intersection point.

Chapter 6, Question #4

An increase in the depreciation rate acts in much the same way as an increase in the population growth rate. More of current savings is required just to keep the amount of capital per capita constant. In equilibrium output per capita and capital per capita decrease.

Chapter 6, Question #5

A destruction of capital.

(a) The long-run equilibrium is not changed by an alteration of the initial conditions. If the economy started in a steady state, the economy will return to the same steady state. If the economy were initially below the steady state, the approach to the steady state will be delayed by the loss of capital.

(b) Initially, the growth rate of the capital stock will exceed the growth rate of the labor force. The faster growth rate in capital continues until the steady state is reached.

(c) The rapid growth rates are consistent with the Solow model's predictions about the likely adjustment to a loss of capital.

Question #2

a) Key equation in Solow growth model

$$k_2 = \frac{sA(k_1)^\alpha}{1+n} + \frac{(1-d)k_1}{1+n}$$

In the steady state, k=k'. So, we can write:

$$k = \frac{sA(k)^\alpha}{1+n} + \frac{(1-d)k}{1+n}$$

$$sA(k^*)^\alpha = (n+d)k^*$$

It is given in the question that *d=1* and *n=0*. So, we have:

$$sA(k^*)^\alpha = k^*$$

$$k = (sA)^{\frac{1}{1-\alpha}}$$

b) It is given that s=0.5; a=0.5; A=10

Thus:

$$k = (sA)^{\frac{1}{1-\alpha}}$$

$$k = (5)^2 = 25$$

Remember the definition of investment:

$$I = K' - (1-d)K$$

Since d=1, we have:

$$I = K' = 25$$

c) Remember $k_2 = \frac{sA\left(k_1\right)^\alpha}{1+n} + \frac{\left(1-d\right)k_1}{1+n}$. Since *n=0* and *d=1*, we have:

$$k_2 = sA\left(k_1\right)^\alpha$$

It is given that s=0.5; a=0.5; A=10; $k_1 = 10$. Thus, we have:

$$k_2 = 5\left(10\right)^{0.5}$$
$$= 15.81$$

Similarly, we can calculate:

$$k_3 = 5\left(15.81\right)^{0.5}$$
$$= 19.88$$

It is easy to calculate the growth rates now...

d) Now we have $k_2 = \frac{sA(k_1)^{\alpha}(1-\tau)}{1+n} + \frac{(1-d)k_1}{1+n}$. Since *n=0* and *d=1*, we have:

$$k_2 = sA(k_1)^{\alpha}(1-\tau)$$

It is given that s=0.5; a=0.5; A=10; $k_1 = 10$. Thus, we have:

$$\begin{aligned} k_2 &= 5(10)^{0.5} \times (0.75) \\ &= 11.86 \end{aligned}$$

Similarly, we can calculate:

$$\begin{aligned} k_3 &= 5(11.86)^{0.5} \times (0.75) \\ &= 12.91 \end{aligned}$$

It is easy to calculate the growth rates now...

Question #3

a) Key equation in Solow growth model

$$k_2 = \frac{sA(k_1)^\alpha}{1+n} + \frac{(1-d)k_1}{1+n}$$

In the steady state, k=k'. So, we can write:

$$k = \frac{sA(k)^\alpha}{1+n} + \frac{(1-d)k}{1+n}$$

$$sA(k^*)^\alpha = (n+d)k^*$$

It is given in the question that *n=0*. So, we have:

$$sA(k^*)^\alpha = dk^*$$

$$k^* = \left(\frac{sA}{d}\right)^{\frac{1}{1-\alpha}}$$

b) It is given that   s=0.4; a=0.5; A=1; d=0.2

Thus:

$$k^* = \left(\frac{sA}{d}\right)^{\frac{1}{1-\alpha}} = \left(\frac{0.4}{0.2}\right)^2 = 4$$

Remember the definition of investment:

$$I = K' - (1-d)K$$

Since d=0.2, we have:

$$I = 4 - (1-0.2)4$$
$$= 0.8$$

c) It is given that s=0.8; a=0.5; A=1; d=0.1

Thus:

$$k^* = \left(\frac{sA}{d}\right)^{\frac{1}{1-\alpha}} = \left(\frac{0.8}{0.1}\right)^2 = 64$$

d) It is given that s=0.5; a=0.5; A=10; d=0.25

Thus:

$$k^* = \left(\frac{sA}{d}\right)^{\frac{1}{1-\alpha}} = \left(\frac{5}{0.25}\right)^2 = 400$$

e) Remember $k_2 = \frac{sA(k_1)^\alpha}{1+n} + \frac{(1-d)k_1}{1+n}$. Since *n=0*, we have:

$$k_2 = sA(k_1)^\alpha + (1-d)k_1$$

It is given that s=0.5; a=0.5; A=10; d=0.25; $k_1 = 300$. Thus, we have:

$$\begin{aligned} k_2 &= 5 \times (300)^{0.5} + (0.75) \times 300 \\ &= 311.60 \end{aligned}$$

Similarly, we can calculate:

$$\begin{aligned} k_3 &= 5 \times (311.60)^{0.5} + (0.75) \times 311.60 \\ &= 321.96 \end{aligned}$$

f) It is easy to calculate the investment and growth rates now...