Testing for Affiliation in First-Price Auctions Using Entry Behaviors

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Abstract

First introduced by Milgrom and Weber (1982), affiliation has been widely used and has become an important concept in the development of auction theory. It has also been used to model bidders' dependence in the structural analysis of auction data. It is thus important to test affiliation among bidders. We show that affiliation among potential bidders' private information (either private values or entry costs) leads to affiliation among their entry decisions. We propose a test for affiliation among potential bidders' private information based on the implication of affiliation on the entry behaviors, which is general and widely applicable to various scenarios. The test is implemented using the simulation based method. We then apply our method to timber sales in Oregon Department of Forestry and find a small but strongly significant level of affiliation among all timber companies.

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1 Introduction

First introduced by Milgrom and Weber (1982), affiliation has been widely used and has become an important concept in the development of auction theory. In the seminal paper by Milgrom and Weber (1982), affiliation is the crucial assumption of the main results in the paper. It implies the existence of a symmetric, increasing, pure strategy equilibrium for the first price auction and is the basis of the well known revenue ranking, that is under affiliation, the English auction has a higher expected revenue than the second price auction which in turn has a higher expected revenue than the first price auction. This revenue ranking result for affiliated bidders is in sharp contrast to the celebrated revenue equivalence result for independent bidders established by Vickrey (1961). Furthermore, in Rodriguez (2000) affiliation implies uniqueness of a monotone equilibrium in a two-person first-price auction with symmetric or asymmetric bidders. While recent work (e.g. Monteiro and Moreira (2006) and de Castro (2007)) has shown that for the existence and uniqueness of the monotone pure strategy equilibrium, affiliation is somewhat stronger and can be relaxed, the revenue ranking crucially relies on the affiliation assumption.[1] Furthermore, two standard implications derived under the assumption of the independent private signals do not hold any more when bidders' private signals are affiliated. First, the effect of the number of potential bidders on bids is not always positive with affiliation as in the IPV model due to the "affiliation effect" (Pinkse and Tan (2005)). Second, while the optimal reserve price under the IPV framework is always higher than the true value of the seller and independent of the number of bidders (Riley and Samuelson (1981), Myerson (1981)), it converges to the true value of the seller as the number of bidders increases within the APV paradigm as shown in Levin and Smith (1996).

Affiliation has also played a key role in the structural analysis of auction data, which was started with Paarsch (1992) who tests the independent private value (IPV) model against the common value (CV) model, which are two polar cases of the affiliated value (AV) model studied in Milgrom and Weber (1982). Since the structural approach is to estimate an econometric model that is closely derived from theory, the assumption made on the underlying information structure is a key to the validity of the structural model. From an econometric point of view, failure in identifying affiliation

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[1] While de Castro (2007) derives a similar revenue ranking under a weaker assumption, it holds in a weaker sense in that it holds on average with respect to all functions of a set under consideration. As a result, for a specific density function in the set, the revenue ranking may break down. Under affiliation, however, for each affiliated density function, the revenue ranking always holds.

among bidders may cause biased estimates of the structural parameters and result in misleading policy conclusions. For instance, one could overestimate bidders' private values when adopting the IPV paradigm for an actual affiliated private value (APV) model (Li, Perrigne, and Vuong (2002)). In the empirical work using the structural approach, affiliation has been assumed by several studies. See, e.g. Li, Perrigne and Voung (2000) for the conditional independent private information (CIPI) model of OCS wildcat auctions, Li, Paarsch and Hubbard (2007) for the APV model of highway procurement auctions in Michigan Department of Transportation. Therefore, it is both interesting and important to test for the affiliation among bidders using field data.

In this paper, we propose a simple approach to test for affiliation among the private information of bidders. Our test builds on the affiliated value model of Wilson (1977) and Milgrom and Weber (1982) and take into account entry and endogenous participation. As is evidenced from the recent work (e.g., Bajari and Hortaçsu (2003), Athey, Levin and Seria (2004), Li and Zheng (2005, 2007), and Krasnokutskaya and Seim (2007)), entry is an important feature in auctions that should be taken into account in empirical analyses of auction data. We show that affiliation among potential bidders' private information (either private values or entry costs) leads to affiliation among their entry decisions. Our test is then proposed based on this implication. From an econometric viewpoint, since our test is based on capturing affiliation through potential bidders' entry behaviors and auction data under consideration usually resemble (unbalanced) panel data settings, we propose a simulated maximum likelihood estimation procedure extending the GHK methods developed by Geweke (1991), Börsch-Supan and Hajivassiliou (1993), and Keane (1994) to our setting. It is worth noting that our approach allows not only for testing for affiliation, but also for testing for asymmetry among potential bidders. It is thus general and flexible, and can be applied to applications of various scenarios.

To the best of our knowledge, our paper is the first one in the literature that proposes a test for affiliation among potential bidders' private information using potential bidders' entry behaviors. The recent work by de Castro and Paarsch (2008) and Jun, Pinkse and Wan (2008) has proposed to use bids to test for affiliation in auctions without entry. Notably, Jun, Prinkse and Wan (2008) apply their test to three data sets from the Offshore Continental Shelf (OCS), California Department of Transport, and Russian Federal Subsoil Resources Management Agency, and find that in most of the cases they cannot reject affiliation between bids and the number of bidders. They interpret this as most likely driven by endogenous entry, highlighting the importance of taking entry into account when testing for affiliation. It is worth noting that testing for affiliation using information from observed bids in a general framework with entry is complicated for several reasons. First, even for a symmetric AV model without entry, Laffont and Voung (1996) show that the private information cannot be identified without additional restrictions. Thus it is difficult to assess the validity of the affiliation assumption using a structural approach. Second, when entry is introduced to the analysis, using observed bids only to test for affiliation becomes challenging as observed bids are now from actual bidders who enter the auction and submit bids, while the affiliation is presumably an assumption made about the dependence structure across all potential bidders' private information that can affect both entry and bidding. The literature on testing implications from auction models, which has been developed since Hendricks and Porter's pioneering work in 1980's (e.g. Porter (1995) for a survey of the work by Hendricks and Porter), on the other hand, has mainly focused on testing between private values and common values. For example, Paarsch (1992) estimates the structural models with a set of specifications within the IPV and CV paradigms and compares the goodness-of-fit of the specifications. Haile, Hong, and Shum (2003) test common values using an insight from the effect of the winner's curse on equilibrium bids. Hendricks, Pinkse and Porter (2003) propose a test for common values when there is a binding reserve price and using bids and the *ex post* realization of the auctioned object's value. The insight that is used in our testing procedure is that affiliation assumed in the joint distribution of private values or the joint distribution of entry costs implies affiliation among their entry decisions. Therefore an affiliation test can be proposed based on the potential bidders' entry behaviors without using bidding information.

Since our approach is based on the testable implication of affiliation on entry behaviors, the observed entry behavior is a key in our approach. While auctions with entry have been studied since 1980's (e.g., Levin and Smith (1994), Samuelson (1985), among others), the empirical analysis of auction data with entry has only drawn considerable attention recently. Several studies have attempted to analyze auction data taking endogenous participation into account. See, e.g. Bajari and Hortacsu (2003) for internet auctions with entry using data from eBay.com, Athey, Levin and Seira (2004) for entry and bidding patterns in sealed bid and open auctions with heterogeneous bidders, Krasnokutskaya and Seim (2007) for the bid preference program and participation in California highway procurement, De Silva, Kosmopoulou and Lamarche (2007) for the effect of information release on entry and survival in procurement auctions, Li and Zheng (2005) for entry and competition effects in procurement auctions, and Li and Zheng (2007) for evaluating how bidders make entry and bidding decisions in Michigan timber auctions. Our paper contributes to this part of the literature by exploiting the implication of affiliation of private values or of entry costs among potential bidders to form an intuitive and easy-to-implement test for affiliation. On the other hand, a caveat of our test is that it is a reduced form in nature, and a confirmation of affiliation from the test can only indicate that there is an affiliation among potential bidders' private information. A further analysis such as a structural analysis should be used to disentangle the driving force for the affiliation of entry behaviors, and to measure the extent to which private values or/and entry costs are affiliated.

We demonstrate our test by applying the test to timber auctions organized by Oregon Department of Forestry (hereafter ODF). We find that bidders are affiliated. Also bidders are asymmetric in the sense that the hauling distance is highly significant in bidders' entry decisions. Our empirical application in timber auctions is interesting in its own right, in addition to serving as an illustration to our proposed testing procedure. Timber auctions have been studied extensively in the empirical literature, because the various auction formats have been used and provided researchers a ground for testing theory and comparing revenues from different auction formats, and also because of the richness of timber auction data in the U.S. and elsewhere.[2] Most of the (structural) empirical work in studying timber auctions, however, has used the IPV paradigm. We find through using our test in the ODF data that the affiliation is significant though at a relatively low level, which turns out to be the first result in the literature to assess affiliation in timber auctions.

This paper is organized as follows. Section 2 describes a general AV model with entry and derive an implication of affiliation on entry behaviors, which is the theoretical foundation of our test. The testing procedure is proposed in Section 3. In Section 4, we apply our test to timber sales organized by ODF. Section 5 concludes.

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[2] The work includes Paarsch (1997), Baldwin, Marshall and Richard (1997), Brannman and Froeb (2000), Haile (2001), Athey and Levin (2001), Li and Perrigne (2003), Haile, Hong and Shum (2003), Athey, Levin and Seira (2004), Li and Zheng (2007), among others.

2 The Theoretical Framework

In this section we consider a first-price sealed-bid auction within the AV paradigm with a public reserve price. To fix ideas, we focus on the symmetric AV model, which is developed by Milgrom and Weber (1982). A single object is auctioned off to N risk-neutral potential bidders who acquire their private information prior to the entry. Affiliation is a terminology describing the dependence among random variables, which is equivalent to multivariate total positivity of order 2 (MTP_2) in the multivariate statistics literature (Karlin and Rinott (1980)). Here we use the same definition of affiliation as in Milgrom and Weber (1982).

Definition. Let y and y' be any two values of a vector of random variables $Y \subseteq \mathbb{R}^n$ with a density $f(\cdot)$. It is said that all elements of Y are affiliated if $f(y \vee y') f(y \wedge y') \geq f(y) f(y')$, where $y \vee y' = (\max\{y_1, y'_1\}, \ldots, \max\{y_n, y'_n\})$ and $y \wedge y' = (\min\{y_1, y'_1\}, \ldots, \min\{y_n, y'_n\})$.

Intuitively, affiliation means that large values for some of the components in Y make other components more likely to be large than small. We also assume that there is an entry cost or a binding reserve price, which may deter some bidders with low private signals from the participation and thus endogenizes the number of actual bidders. We denote the utility of bidder i from the object by $U_i = u_i(S, V)$, where V is a n-dimensional vector with support of $[\underline{v}, \overline{v}]$, whose the i-th element V_i represents private signal of bidder i, and S is an m-dimensional vector representing additional information.[3] Furthermore, $u_i(S, V) = u(S, V_i, \{V_j\}_{j \neq i})$ in the symmetric case as assumed in Milgrom and Weber (1982). Denote $f(s, v)$ the joint probability density of S and V. We make the following assumption as in Milgrom and Weber (1982).

Assumption. $S_1, \cdots S_m, V_1, \cdots V_n$ are affiliated.

A binding reserve price r or a non-trivial entry cost k or both keeps bidders with low private signals away from the auction. Potential bidders make their entry decisions according to their private signals. As is shown by Milgrom and Weber (1982), in both cases with symmetric bidders the entry decision is governed by a screening level, v^*. When only a public binding reserve price is present and there is no entry cost, the screening level is defined as

$$v^* = \inf\{v_c | E[U_c | V_c = v_c, V_j < v_c, j \neq c] \geq r\}, \tag{1}$$

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[3] Such a setup nests both IPV paradigm and CV paradigm. See Milgrom and Weber (1982) for detail.

where subscript c indexes bidder c. When both public reserve price and entry costs are present, v^* is defined as

$$v^* = \inf \left\{ v_c | E \left[(U_c - r) 1 (V_j < v_c, j \neq c) | V_c = v_c \right] \geq k \right\}. \tag{2}$$

As a result, only those potential bidders whose private signals are above the screening levels become actual bidders and submit bids.[4] Actual bidder i submits his bid to maximize his expected profit, which can be written as

$$E \left[(U_i - b_i) 1 (b_j < b_i, j \neq i) | V_i = v_i \right], \ v_i \in [v^*, \overline{v}]. \tag{3}$$

As in the literature, we focus on an increasing differentiable Bayesian Nash equilibrium strategy $b_i = s(v_i)$ which is characterized by the first order condition of the maximization problem (3) with respect to b_i with boundary conditions $s(v^*) = r$.

Note that the entry behavior of each potential bidder in an auction is determined by the cut-off point v^*. It is then natural to think of affiliated bidders as more likely to have similar entry behaviors. Suppose that two bidders are affiliated and bidder 1 has high private value, which is above v^*. By the definition of affiliation, it is more likely that bidder 2 has high private value as well. Therefore the probability that both bidder 1 and 2 participate in the auction should be no less than the probability that bidder 1 participates and bidder 2 does not. The following proposition formalizes this intuition.

Proposition 1. Let $D = (D_1, \ldots, D_N) \in \{0, 1\}^N$ denote bidder $1, \cdots$, bidder $N's$ entry decisions. If $V_1, \ldots, V_N$ are affiliated, then $D_1, \ldots, D_N$ are also affiliated.

Proof: Since the entry behavior of bidder i is controlled by the screening level v^* defined in equation (1) or (2), D_i is then defined as $D_i = 1(V_i > v^*)$ and is a non-decreasing function of V_i. Following Theorem 3 in Milgrom and Weber (1982) and given that $V_1, \ldots, V_N$ are affiliated, $D_1, \ldots, D_N$ are affiliated.

Proposition 1 implies that in the case where potential bidders first draw their private values before entering, affiliation in potential bidders' private values lead to affiliation in their entry

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[4] This entry model is the same as the one proposed by Samuelson (1985) in nature, in which potential bidders adopt pure strategy in their entry decision. An alternative entry model is developed by Levin and Smith (1994), in which bidders make entry decision before learning their private information and therefore, their entry behaviors are randomized, and under the symmetry assumption, each bidder has the same probability of participation.

decisions. Note that in this case, it is assumed that potential bidders have the same entry cost, and the entry cost can be interpreted as related to bid preparation. Alternatively, potential bidders can be assumed to have heterogenous entry costs, and do not draw their private values until after entering the auction.[5] In this case, entry costs, which can involve costs for both information acquisition and bid preparation, are different across potential bidders and are also part of bidders' private information. Furthermore, in this case, the cut-off point for the entry decision is the ex ante expected payoff for a potential bidder from entering. A potential bidder enters the auction if his entry cost is below the cut-off point, and otherwise stays out. In this case, it can be shown using the same argument as in proving Proposition 1 that if potential bidders' entry costs are affiliated, then their entry decisions are affiliated whether or not the potential bidders' private values are affiliated.

3 The Affiliation Test

The insight we gain from the previous section is that affiliation among potential bidders' private information (either private values or entry costs) leads to affiliation among their entry decisions. Therefore we propose to test for affiliation among bidders' private information by testing for affiliation among their entry decisions. Since in practice we usually deal with a large number of (heterogenous) auctions, we assume that we observe a $1 \times k_1$ auction-specific covariate vector, denoted by x_ℓ, $\ell = 1, ..., L$, where L is the number of auctions in the data set. For auction ℓ in the data, there are N_ℓ potential bidders. To control for the possible asymmetry among potential bidders, we include a $1 \times k_2$ bidder-specific covariate vector denoted by $z_{\ell i}$, $i = 1, ...N_\ell$. We define $D_{\ell i}$ to be 1 if potential bidder i enters the ℓ-th auction and 0 otherwise. We use a binary choice model to model the entry decision as follows,

$$D_{\ell i} = 1\left(x_\ell \beta + z_{\ell i}\gamma + \eta_\ell + \varepsilon_{\ell i} > 0\right) \tag{4}$$

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[5] This case has been modeled by Li and Zheng (2005, 2007) within the symmetric IPV paradigm, and by Athey, Levin and Seira (2004) and Krasnokutskaya and Seim (2007) for two groups of bidders within the IPV paradigm.

where η_ℓ denotes the auction heterogeneity unobserved by the econometrician,[6] and $\varepsilon_{\ell i}$ denotes idiosyncratic error unobserved by the econometrician, and independent of x_ℓ, $z_{\ell i}$, and η_ℓ. The next proposition links the affiliation among $D_{\ell i}$ to the affiliation among $\varepsilon_{\ell i}$.

Proposition 2. If $\varepsilon_{\ell 1}, \ldots, \varepsilon_{\ell N_\ell}$ are affiliated, then $D_{\ell 1}, \ldots, D_{\ell N_\ell}$ are affiliated.

Proof: It follows the fact that $D_{\ell i}$ is a non-decreasing function of $\varepsilon_{\ell i}$.

Testing for affiliation among $D_{\ell i}$ now amounts to testing for affiliation among $\varepsilon_{\ell i}$ in our setting. To implement the test, we further assume that $\eta_\ell, \varepsilon_{\ell 1}, \ldots, \varepsilon_{\ell N_\ell}$ follow a joint normal distribution with mean $\mu = (0, \ldots, 0)'$ and covariance $\Sigma_{(N_\ell+1)\times(N_\ell+1)} = \begin{bmatrix} \sigma^2 & 0 & \cdots & 0 \\ 0 & 1 & & \rho \\ \vdots & & \ddots & \\ 0 & \rho & & 1 \end{bmatrix}$, implying that η_ℓ is independent of $\varepsilon_{\ell i}$, and $\varepsilon_{\ell i}$ and $\varepsilon_{\ell j}$ have a correlation denoted by ρ, the same across all the potential bidders/auctions. Under our setting, ρ represents the affiliation among $\varepsilon_{\ell 1}, \ldots, \varepsilon_{\ell N_\ell}$. This follows from Sarkar (1969) and Barlow and Proschan (1975) who show that for $X = (X_1, \cdots, X_n) \sim N(0, \Omega)$, $X_1, \ldots, X_n$ are affiliated if and only if the off-diagonal elements of matrix $-\Omega^{-1}$ are nonnegative, which implies, in our case, that $\varepsilon_{\ell 1}, \ldots, \varepsilon_{\ell N_\ell}$ are affiliated if and only if $\rho \geq 0$. As a result, we can construct the following hypotheses to test for affiliation among the private signals of all potential bidders,

$$H_0 : \rho \geq 0,$$
$$H_1 : \rho < 0.$$

A by-product of our setting is that the symmetry assumption can be tested by testing $\gamma = 0$, since the possible asymmetry among potential bidders is accommodated by the bidder-specific variables.

3.1 A Smoothly Simulated Maximum Likelihood Estimator

The problem now becomes estimating the unknown parameters $\theta = (\beta, \gamma, \sigma, \rho)$ in the binary latent variable model. Under the specifications given above, we can write down the joint likelihood

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[6] As shown in Krasnokutskaya (2003), controlling for unobserved auction heterogeneity can be important in applications.

function for all N_ℓ potential bidders at the ℓ-th auction in terms of their entry decisions. For instance, the likelihood of all potential bidders participating in the ℓ-th auction is

$$\begin{aligned} p_\ell(\theta) &= \Pr\left(D_{\ell i}=1, i=1,\ldots,N_\ell\right) \\ &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty} 1\left(\varepsilon_\ell \in A_\ell\right) f\left(\varepsilon_{\ell 1},\ldots,\varepsilon_{\ell N_\ell},\eta_\ell\right) d\varepsilon_{\ell 1}\cdots d\varepsilon_{\ell N_\ell} d\eta_\ell \\ &= E_{\eta_\ell}\left[\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty} 1\left(\varepsilon_\ell \in A_\ell\right) f_\varepsilon\left(\varepsilon_{\ell 1},\ldots,\varepsilon_{\ell N_\ell}\right) d\varepsilon_{\ell 1}\cdots d\varepsilon_{\ell N_\ell}\right], \end{aligned} \tag{5}$$

where A_ℓ denotes the set $\{\varepsilon_\ell | \varepsilon_{\ell i} > -\left(x_\ell\beta + z_{\ell i}\gamma + \eta\right), i=1,\ldots,N_\ell\}$ and the last equality follows from the independence assumption of $\varepsilon_{\ell 1},\ldots,\varepsilon_{\ell N_\ell}$ and η_ℓ. Similarly, we can express the likelihood of entry/non-entry of all potential bidders at all other auctions. In principle, θ can be estimated by MLE with the joint log-likelihood function defined as

$$\pounds\left(D,x,z;\theta\right) = \sum_{\ell=1}^{L} \ln p_\ell\left(\theta\right).$$

There are a couple of complications arising from implementing the MLE here. First, $p_\ell(\theta)$ involves multiple $(N_\ell+1)$ integrations. Therefore when the number of potential bidders N_ℓ is large, the MLE becomes computationally intensive because of the large number of multiple integrals involved. this issue can be addressed through using the simulation based method to approximate the otherwise difficult-to-calculate integrals. Second, with the use of the simulation based method, the objective function may become non-smooth due to the index function $1\left(\varepsilon_\ell \in A_\ell\right)$. To address both issues, we adopt a smooth and unbiased simulator often called the GHK simulator after Geweke, Hajivassiliou, and Keane, which is found to be the most reliable method for simulating normal rectangle probability among others (e.g. Hajivassiliou, McFadden and Ruud (1996)). Specifically, in our case, the covariance matrix of $\varepsilon_{\ell i}$ is $\sum_{N_\ell} = \begin{bmatrix} 1 & & \rho \\ & \ddots & \\ \rho & & 1 \end{bmatrix}_{N_\ell \times N_\ell}$. There must exist a lower triangular matrix h_ℓ such that $h_\ell \cdot h_\ell' = \sum_{N_\ell}$. Then $\varepsilon_\ell = \left(\varepsilon_{\ell 1},\ldots,\varepsilon_{\ell N_\ell}\right)'$ can be written as $\varepsilon_\ell = h_\ell \xi_\ell$, where $\xi_\ell = \left(\xi_{\ell 1},\ldots,\xi_{\ell N_\ell}\right)'$ follows N_ℓ-variate standard normal distribution. Since h_ℓ is a lower triangular matrix, we have the following recursive formula for $\varepsilon_{\ell i}$.

$$\begin{aligned}
\varepsilon_{\ell 1} &= h_{\ell,11}\xi_{\ell 1} \qquad (6)\\
\varepsilon_{\ell 2} &= h_{\ell,21}\xi_{\ell 1} + h_{\ell,22}\xi_{\ell 2}\\
&\vdots\\
\varepsilon_{\ell N_\ell} &= h_{\ell,N_\ell 1}\xi_{\ell 1} + \cdots + h_{\ell,N_\ell N_\ell}\xi_{\ell N_\ell},
\end{aligned}$$

where $h_{\ell,ij} = h_\ell(i,j)$. Then we can make simulation draws based on such a decomposition. For example, the probability that all bidders participate in the ℓ-th auction $p_\ell(\theta)$ in equation (5) can be written as the following form

$$\begin{aligned}
p_\ell(\theta) &= E_v\left[\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty} 1(\xi_\ell \in B_\ell)\, f_\xi\left(\xi_{\ell 1},\ldots,\xi_{\ell N_\ell}\right) d\xi_{\ell 1}\cdots d\xi_{\ell N_\ell}\right]\\
&= E_v\left[\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty} g(x_\ell, z_\ell, \xi_l, v;\theta|v)\, 1(\xi_\ell \in B_\ell)\,\frac{f_\xi\left(\xi_{\ell 1},\ldots,\xi_{\ell N_\ell}\right)}{g(x_\ell, z_\ell, \xi_l, v;\theta|v)} d\xi_{\ell 1}\cdots d\xi_{\ell N_\ell}\right]\\
&= E_v\left[\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty} g(x_\ell, z_\ell, \xi_l, v;\theta|v)\, 1(\xi_\ell \in B_\ell)\,\frac{\phi(\xi_1)\times\cdots\times\phi\left(\xi_{\ell N_\ell}\right)}{g(x_\ell, z_\ell, \xi_l, v;\theta|v)} d\xi_{\ell 1}\cdots d\xi_{\ell N_\ell}\right]\\
&= E_v\left[E\left(g(x_\ell, z_\ell, \xi_l, v;\theta|v)\right)\right]\\
&= E\left(g(x_\ell, z_\ell, \xi_l, v;\theta)\right),
\end{aligned}$$

where B_ℓ is the set derived from A_ℓ according to the transformation (6), and $v = \eta/\sigma$ and $g(x_\ell, z_\ell, \xi_l, v;\theta|v) = \left[1-\Phi\left(\frac{-x_\ell\beta - z_{\ell 1}\gamma - \sigma v}{h_{\ell,11}}\right)\right]\times\cdots\times\left[1-\Phi\left(\frac{-x_\ell\beta - z_{\ell N_\ell}\gamma - \sigma v - h_{\ell,N_\ell 1}\xi_1 - \cdots - h_{\ell,N_\ell N_\ell - 1}\xi_{\ell N_\ell - 1}}{h_{\ell,N_\ell N_\ell}}\right)\right]$. The second last equality follows the fact that $\frac{\phi(\xi_1)\times\cdots\times\phi\left(\xi_{\ell N_\ell}\right)}{g(\xi_l|v)} 1(\xi_\ell \in B_\ell)$ is actually a joint density function of ξ_ℓ conditional v. As a result, $p_\ell(\theta)$ can be approximated by $\widetilde{p}_\ell(\theta) = \frac{1}{T}\sum_{t=1}^{T} g_t(x_\ell, z_\ell, \xi_l, v;\theta)$ and $g_t(x_\ell, z_\ell, \xi_l, v;\theta)$ is simulated by the following procedure.

1. First draw v from $N(0,1)$ and calculate $1-\Phi\left(\frac{-x_1\beta - \sigma v}{h_{11}}\right)$.

2. Draw $\xi_{\ell 1}$ from $N(0,1)$ truncated at $\frac{-x_\ell\beta - z_{\ell 1}\gamma - \sigma v}{h_{\ell,11}}$ from below, and calculate
   $1-\Phi\left(\left.\frac{-x_\ell\beta - z_{\ell 2}\gamma - \sigma v - h_{\ell,21}\xi_{\ell 1}}{h_{\ell,22}}\right|\xi_{\ell 1}\right)$.

$\vdots$

N. Draw $\xi_{\ell N_\ell - 1}$ from $N(0,1)$ truncated at $\frac{-x_\ell\beta - z_{\ell N_\ell}\gamma - \sigma v - h_{\ell,N_\ell 1}\xi_1 - \cdots - h_{\ell,N_\ell N_\ell - 2}\xi_{\ell N-2}}{h_{\ell,N_\ell - 1 N_\ell - 1}}$ and calculate

$$1-\Phi\left(\frac{-x_\ell\beta-z_{\ell N_\ell}\gamma-\sigma v-h_{\ell,N_\ell 1}\xi_1-\cdots-h_{\ell,N_\ell N_\ell-1}\xi_{\ell N-1}}{h_{\ell,N_\ell N_\ell}}\middle|\,\xi_{\ell 1},\ldots,\xi_{\ell N_\ell-1}\right).$$

Define the corresponding simulated likelihood function as $\widetilde{\mathcal{L}}\left(D,x,z;\theta\right)=\sum_{\ell=1}^{L}\ln\widetilde{p}_\ell\left(\theta\right).$ Following Börsch-Supan and Hajivassiliou (1993), a smoothly simulated maximum likelihood estimator (SSMLE) can be proposed as

$$\widehat{\theta}_{SSMLE}=\arg\max_{\theta}\widetilde{\mathcal{L}}\left(D,x,z;\theta\right).$$

In practice, to guarantee that ρ is between -1 and 1 and σ is positive, we make two transformations, namely, $\rho=\frac{2}{1+\exp(\widetilde{\rho})}-1$ and $\sigma=\exp\left(\widetilde{\sigma}\right).$

As the SSMLE is essentially one type of the simulated maximum likelihood estimator, $\widehat{\theta}_{SSMLE}$ has the same asymptotic normal distribution as that of the usual MLE as the simulation number, T, is large enough in the sense that $T/\sqrt{L}\to\infty$. Let $s_l\left(\theta\right)=\nabla_\theta\left(\ln p_l\left(\theta\right)\right)'$, then $\sqrt{L}\left(\widehat{\theta}_{SSMLE}-\theta\right)\to N\left(0,\Lambda\right)$, where $\Lambda=E\left[s_\ell'\left(\theta\right)s_\ell\left(\theta\right)\right]^{-1}$(e.g. Train (2003)).

4 Affiliation in Oregon Timber Auctions

This section applies our testing procedure to the timber sales organized by the ODF from January 2002 to June 2007. All auctions are held as first-price sealed-bid auctions and in a format of scale auctions, in which bidders compete in unit prices.

4.1 Timber Auction Data

Before each sale, ODF "cruises" a selected tract of timber and obtains information of the tract such as the species composition, volume of each species, the quality of timber and so on. After the "cruise", a bid notice is usually released 4-6 weeks prior to the sale, which provides detailed information about the sale, including auction date and location, tracts location, species quality, appraised price, quality grade of timber as well as other related information. Bidders acquire their own private signals through different ways after knowing the sale and decide whether to enter the auction according to their expected profits. The actual bidders submit their bids in a sealed envelope. The sale is awarded to the bidder with the highest bid provided that it is above the reserve price.

In the data, there are 415 sales held in 5 regions of Oregon State and 1501 observed bids in total. There is only one bid species in most sales, whereas some sales consist of two or more bid species, which are deleted from our sample due to the "skewed bidding" issue discussed in Athey and Levin (2001). This reduces the sample sizes of sales and bids down to 324 and 1257, respectively. Among the remaining sales, the dominant bid species is Douglas-fir, which accounts for 91.98%. To make the auctions as homogeneous as possible, all auctions with other bid species are removed from data. Some sales which have missing values are removed too. The final numbers of auctions and observed bids are 282 and 1139, respectively.

For each auction we observe date of sale opening, type of work, region of sale, appraised price, appraised volume, length of the contract, diameter of breast height (DBH), and quality grade as well. The appraised price is actually the reserve price of the auction. The volume and the length of the contract represent the size of the sale. Both DBH and quality grade are variables representing the quality of timber. Quality grade is not observed directly. The bid species of a sale often consists of a mixture of several grades of quality. We use $1, 2, \ldots,$ up to 18 to denote the letter-grades used by ODF. Grade is then constructed by a weighted average of grades using the volumes of each grade as the weight.

Since our test is based on the entry behavior, the identities of potential bidders are crucial to constructing the entry behaviors. In the timber auctions studied in this paper, however, we cannot directly observe the potential bidders. In some procurement auctions, information on firms who have requested bidding proposals is available, in which case Li and Zheng (2005) suggest to consider those who have requested bidding proposals as potential bidders. We do not have this information in our data, as is usually the case for timber auctions. Therefore, we follow Athey, Levin and Seira (2004) and Li and Zheng (2007) to construct potential bidders. Specifically, we first divide all sales in the original data set into 146 groups each of which contains all sales held in the same district in the same quarter of the same year. The potential bidders of a sale are then all bidders who submit at least one bid in the sales of that group. That is, all auctions in the same group have the same set of potential bidders. Note that in constructing the potential bidders we use the original data set including all auctions with two or more bid species, which have been removed from the final data set we study.

Hauling distance has been shown to be significant in affecting bidders's bidding decision (e.g.

Brannman and Froeb (2000)). It should be controlled for when studying potential bidders' entry decisions as well. However we do not observe the distance directly. Instead we observe the location of tract of each sale and address of each bidders. To construct the hauling distance we first transfer the location of tract into latitude and longitude through Oregon Latitude and Longitude Locator (OLALO) and then calculate the distance between the tract and potential bidders through Google Maps. To consider the potential difference among auctions which were held in different regions and had different types of work, we construct two regional dummies and two type dummies according to three regions and three types of work observed in the data. Summary statistics of the data are given in Table 1.[7]

Since we will focus on the entry behaviors of the potential bidders to infer affiliation, we highlight the histogram of the entry proportion of the data in Figure 1, where the entry proportion is defined as the ratio between the number of the actual bidders and the number of potential bidders. Figure 1 presents some interesting features. As one can see from this figure, the auctions which have more than 90% entry proportion have the largest frequency. The number of auctions which have less than 30% or greater than 70% entry proportions is 160, which is a bit above a half of the total number of auctions in the data set. This can be viewed as an indication of a small level of affiliation among potential bidders.

4.2 Results

Table 2 presents the estimation results, which are obtained through the SSMLE with $T = 100$. Since our primary goal is to test for affiliation, we are mainly interested in the estimate of ρ, which turns out to be 0.2281 with a standard error equal to 0.0493. This result suggests that ρ is significant and therefore bidders are affiliated. But the magnitude of ρ indicates that the level of affiliation is not high, though significant. This is consistent with the finding through the simple analysis of the entry proportion. The standard deviation of the unobserved heterogeneity is quite small and insignificant, which implies that the variables we use to control for auction heterogeneity have captured most of the auction heterogeneity that affects potential bidders' entry decision. Another interesting finding is that the hauling distance matters in the entry behaviors and one percentage

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[7] Entry behavior is a binary variable. It is equal to 1 for a potential bidder of an auction if we observe his bid in that auction in our data set and 0 otherwise.

increase in the hauling distance will decrease the probability of entry by about 8.75% on average. As a result the hypothesis of symmetric bidders is rejected. This means that the hauling distance not only affects bidders' bidding behaviors as found in Brannman and Froeb (2000), but also affects potential bidders' entry decisions. As far as auction-specific covariates are concerned, except that the duration of a contract and DBH are not significant, the estimates of all other covariates are intuitive. Potential bidders are more likely to enter the auction with higher volume or higher quality, as suggested by the positive coefficients of the log volume and log grade. On the other hand, the negative coefficient of the number of potential bidders implies that the competition may deter potential bidders' participation, which is consistent with the theoretical results in the literature (e.g. Li and Zheng (2005) for the relationship between the entry probability and the number of potential bidders).

5 Conclusions

Affiliation is an important assumption that has been used in both theoretical and empirical framework in studying auctions. It is thus an important econometric issue as to how to test for affiliation using field data. In this paper, we propose a novel approach to test for affiliation. Using observed bids to test for affiliation is difficult when endogenous entry is present. We circumvent this problem by using only potential bidders' entry behaviors based on the insight that affiliation among potential bidders' private information implies the affiliation among potential bidders' entry behaviors.

Our testing procedure requires estimation of a multivariate probit model. It is general as it can accommodate various scenarios such as asymmetric bidders, public/secret reserve prices, and unobserved auction heterogeneity. We propose the SSMLE to estimate the model as the SSMLE overcomes computational complexity associated with the estimation and makes the estimation computationally tractable. It is worth noting that while our primary goal is to test for affiliation, our testing procedure can also be used to verify some other assumptions such as symmetry among bidders.

We apply our approach to the timber auctions held by ODF. Our results indicate that the affiliation among potential bidders' entry decisions is significant but of a small level. Furthermore, the potential bidders' entry decisions are affected by the hauling distance, meaning that the potential bidders are heterogenous. These findings offer insight and guidance in terms of structural modeling, as in view of these results, a structural model that is used to study the timber auctions organized by ODF should take into account the affiliation among bidders' private values or/and bidders' entry costs, and asymmetry among potential bidders. This is considered in Li and Zhang (2008).

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Table 1: Summary Statistics

	Observation	Mean	Std. Dev.
Entry Behavior	2055	0.5543	0.4972
Distance	2055	78.8476	47.1271
Volumn	282	3256.7520	2622.2350
Duration	282	781.5390	225.6230
Grade	282	10.2935	0.4549
DBH	282	16.5670	4.8455
# of Poten. Bidders	282	7.2872	2.9897
Region 1	282	0.8262	0.3796
Region 2&3	282	0.1596	0.3669
Clear, Recovery	282	0.4007	0.4909
Combo, Recovery	282	0.2731	0.4463

Table 2: Estimation Results

	Estimates	Std. Error
Log of Volume	0.1496*	0.0732
Log of Duration	0.014	0.1535
Log of Grade	5.3086*	2.3448
Log of DBH	−0.3006	0.3863
Potential Bidder	−0.0755*	0.0158
ρ^{**}	0.2281*	0.0493
σ^{**}	0.1560	3.6562
Log of Distance	−0.2064*	0.0223

$*$ denotes 5% significance.

$**$ The estimates and standard errors are the transformed ones.

Figure 1: Histogram of Entry Proportion